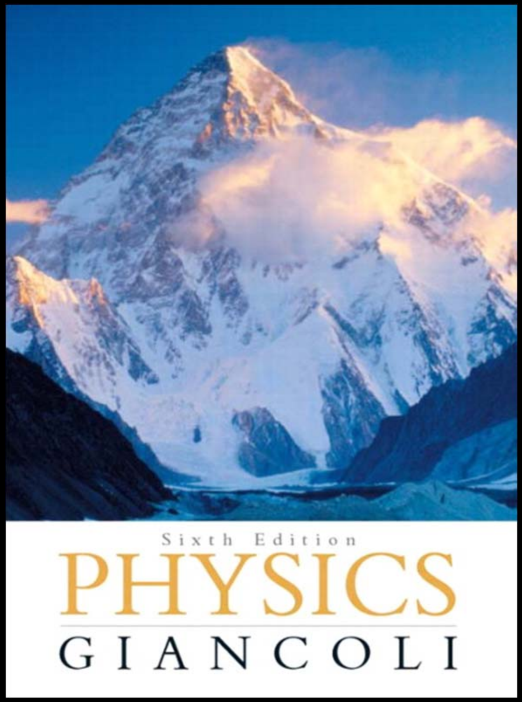


Lecture PowerPoint

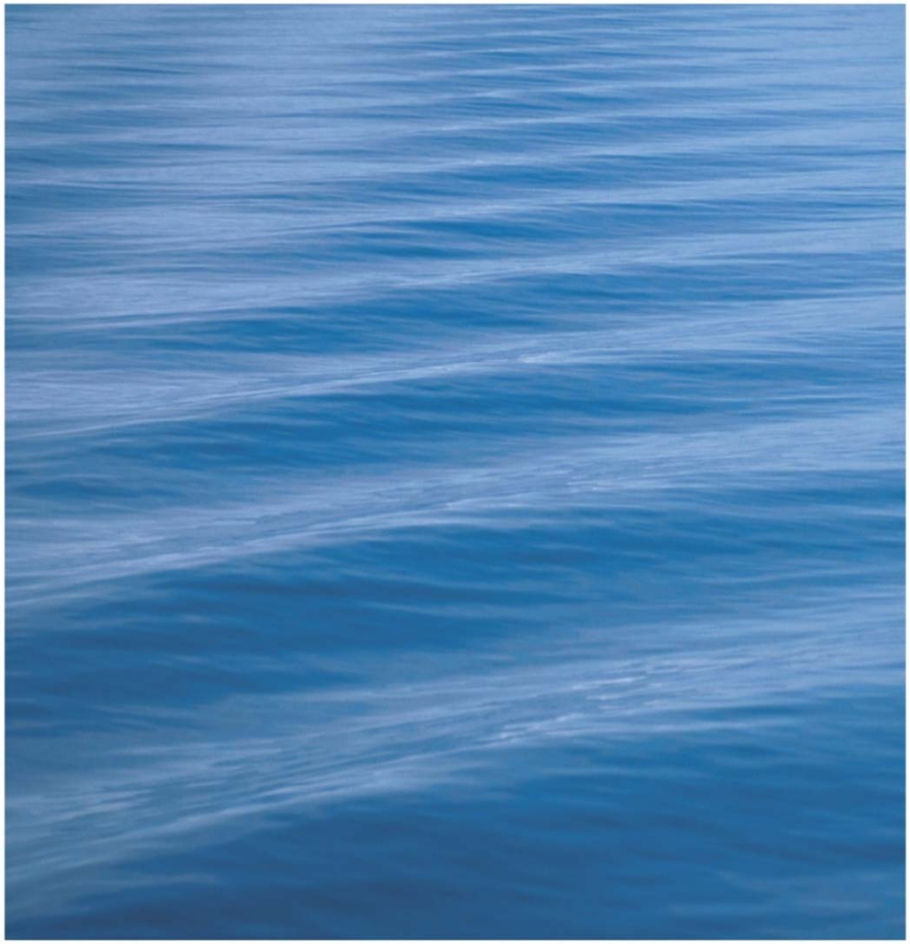
Chapter 11

Physics: Principles with Applications, 6th edition

Giancoli

Chapter 11

Vibrations and Waves



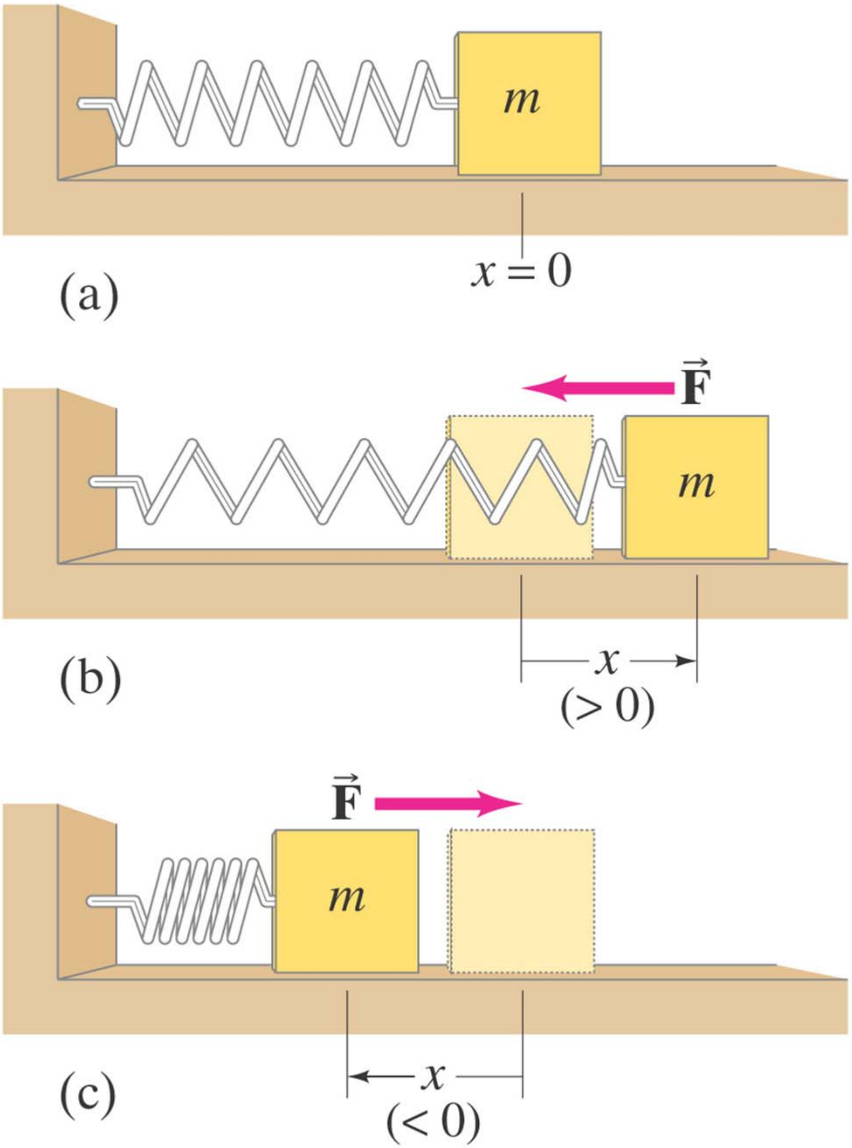
Units of Chapter 11

- Simple Harmonic Motion
- Energy in the Simple Harmonic Oscillator
- The Period and Sinusoidal Nature of SHM
- The Simple Pendulum
- Damped Harmonic Motion
- Forced Vibrations; Resonance
- Wave Motion
- Types of Waves: Transverse and Longitudinal

Units of Chapter 11

- Energy Transported by Waves
- Intensity Related to Amplitude and Frequency
- Reflection and Transmission of Waves
- Interference; Principle of Superposition
- Standing Waves; Resonance
- Refraction (Definition only)
- Diffraction (Definition only)

11-1 Simple Harmonic Motion



If an object vibrates or oscillates back and forth over the same path, each cycle taking the same amount of time, the motion is called periodic. The mass and spring system is a useful model for a periodic system.

11-1 Simple Harmonic Motion

We assume that the surface is frictionless. There is a point where the spring is neither stretched nor compressed; this is the equilibrium position. We measure displacement from that point (x = 0 on the previous figure).

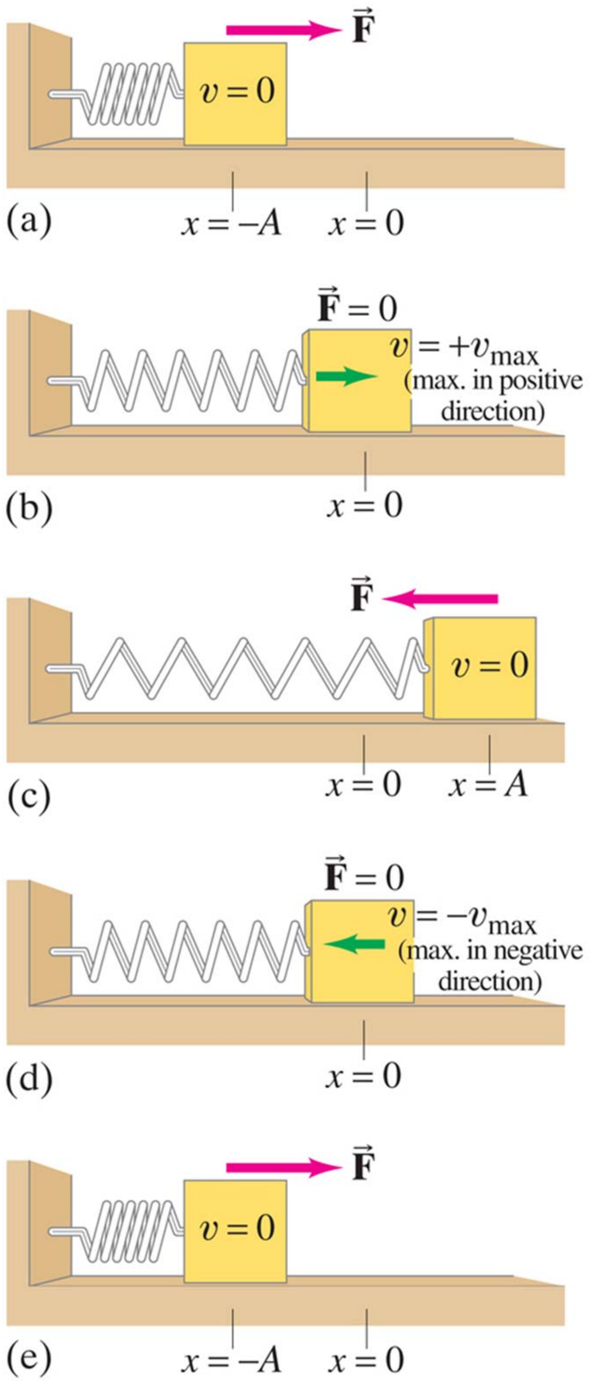
The force exerted by the spring depends on the displacement:

$$F = -kx \qquad (11\text{-}1)$$

11-1 Simple Harmonic Motion

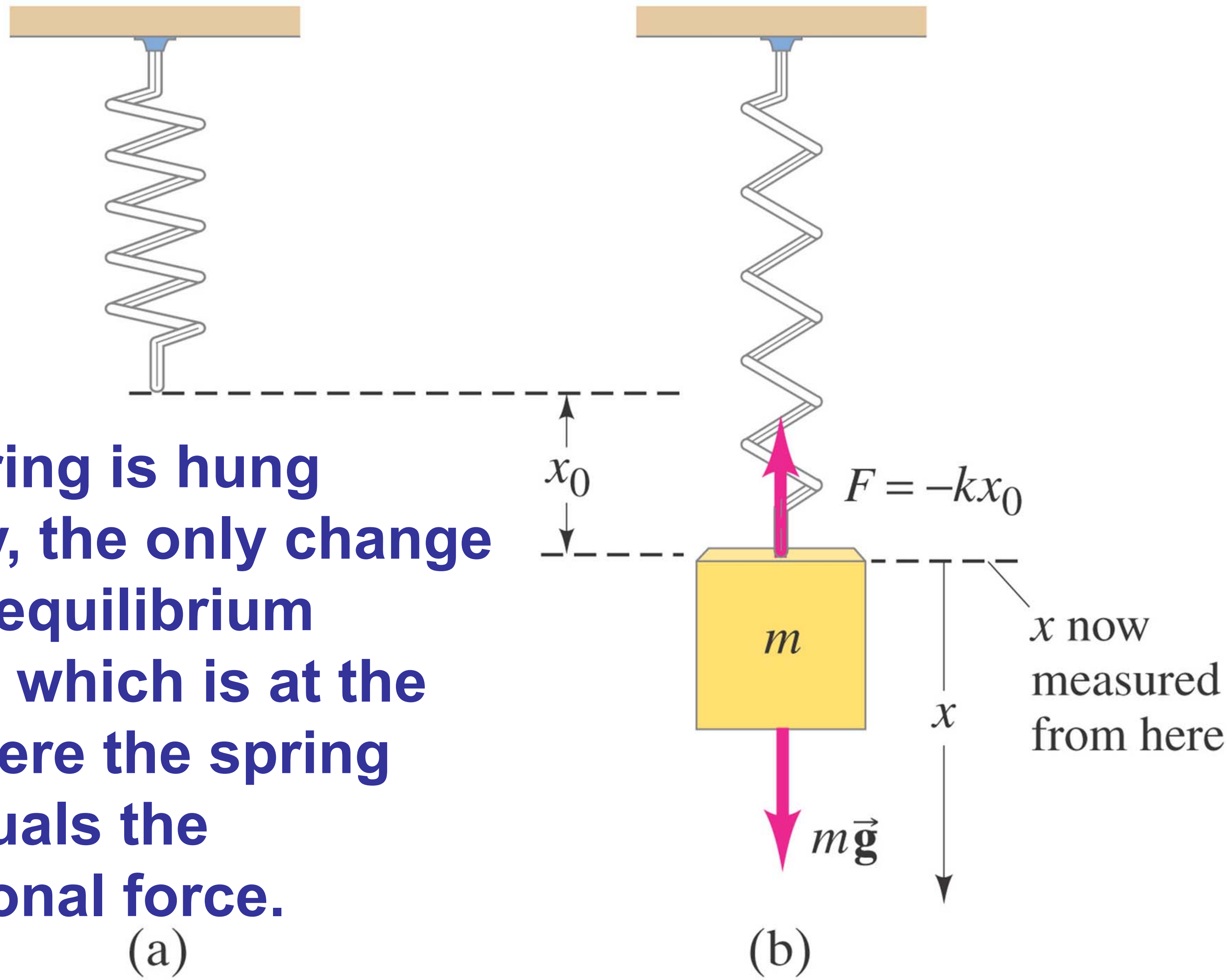
- The minus sign on the force indicates that it is a restoring force – it is directed to restore the mass to its equilibrium position.
- *k* is the spring constant
- The force is not constant, so the acceleration is not constant either

11-1 Simple Harmonic Motion



- Displacement is measured from the equilibrium point
- Amplitude is the maximum displacement
- A cycle is a full to-and-fro motion; this figure shows half a cycle
- Period is the time required to complete one cycle
- Frequency is the number of cycles completed per second

11-1 Simple Harmonic Motion



If the spring is hung vertically, the only change is in the equilibrium position, which is at the point where the spring force equals the gravitational force.

11-1 Simple Harmonic Motion

- Any vibrating system where the restoring force is proportional to the displacement from equilibrium is in simple harmonic motion (SHM), and is often called a simple harmonic oscillator.
- Any motion which can be described as a sinusoid is SHM.
- SHM is a *projection* of circular motion.

11-2 Energy in the Simple Harmonic Oscillator

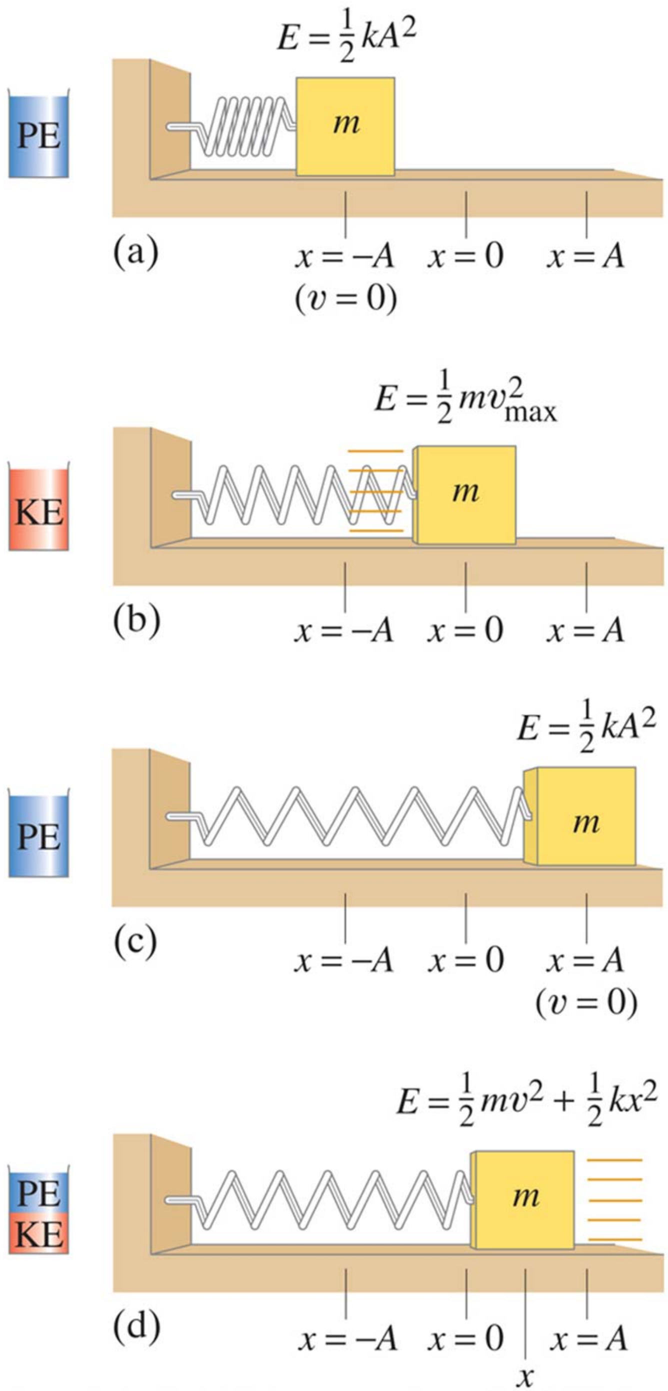
We already know that the potential energy of a spring is given by:

$$\text{PE} = \tfrac{1}{2}kx^2$$

The total mechanical energy is then:

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \qquad \textbf{(11-3)}$$

The total mechanical energy will be conserved, as we are assuming the system is frictionless.



11-2 Energy in the Simple Harmonic Oscillator

If the mass is at the limits of its motion, the energy is all potential.

If the mass is at the equilibrium point, the energy is all kinetic.

We know what the potential energy is at the turning points:

$$E = \frac{1}{2}kA^2 \qquad \textbf{(11-4a)}$$

11-2 Energy in the Simple Harmonic Oscillator

The total energy is, therefore $\frac{1}{2}kA^2$

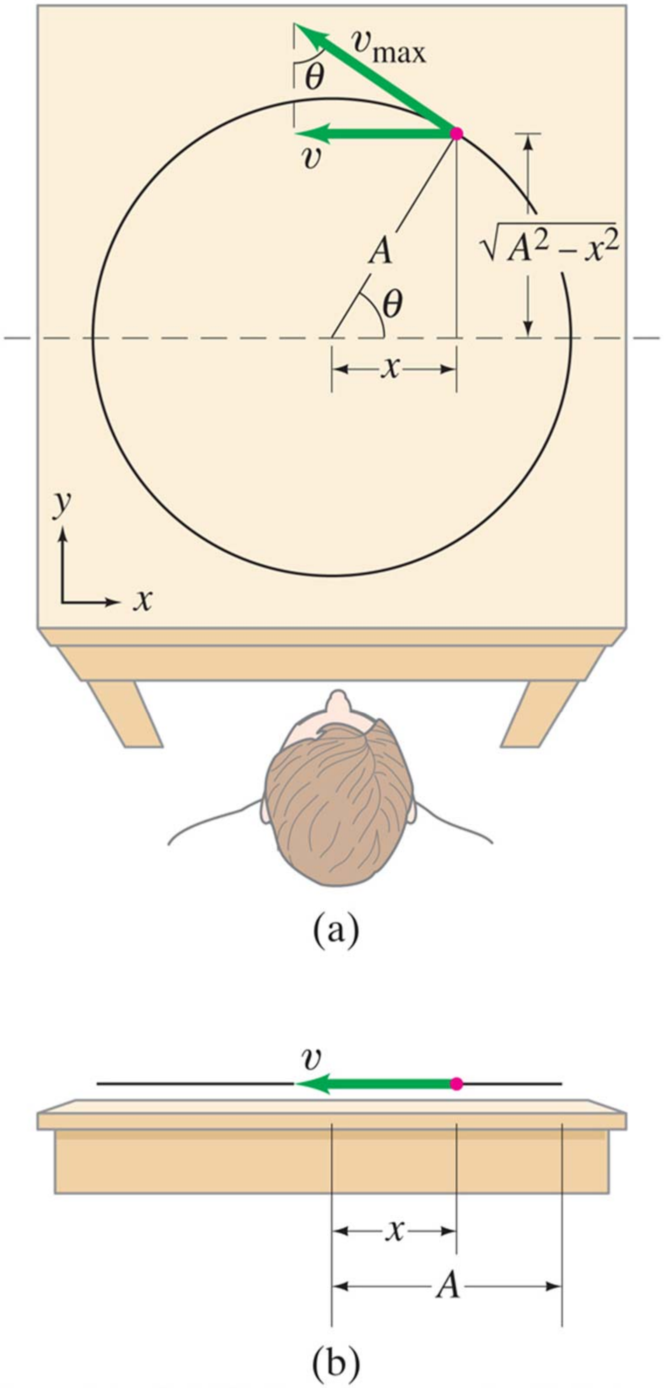
And we can write:

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \qquad \textbf{(11-4c)}$$

This can be solved for the velocity as a function of position:

$$v = \pm v_{\max}\sqrt{1 - \frac{x^2}{A^2}} \qquad \textbf{(11-5)}$$

where $v_{\max}^2 = (k/m)A^2$



11-3 The Period and Sinusoidal Nature of SHM

If we look at the projection onto the *x* axis of an object moving in a circle of radius *A* at a constant speed v_{max}, we find that the *x* component of its velocity varies as:

$$v = v_{max}\sqrt{1 - \frac{x^2}{A^2}}$$

This is identical to SHM.

11-3 The Period and Sinusoidal Nature of SHM

Therefore, we can use the period and frequency of a particle moving in a circle to find the period and frequency:

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \textbf{(11-7a)}$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \textbf{(11-7b)}$$

11-3 The Period and Sinusoidal Nature of SHM

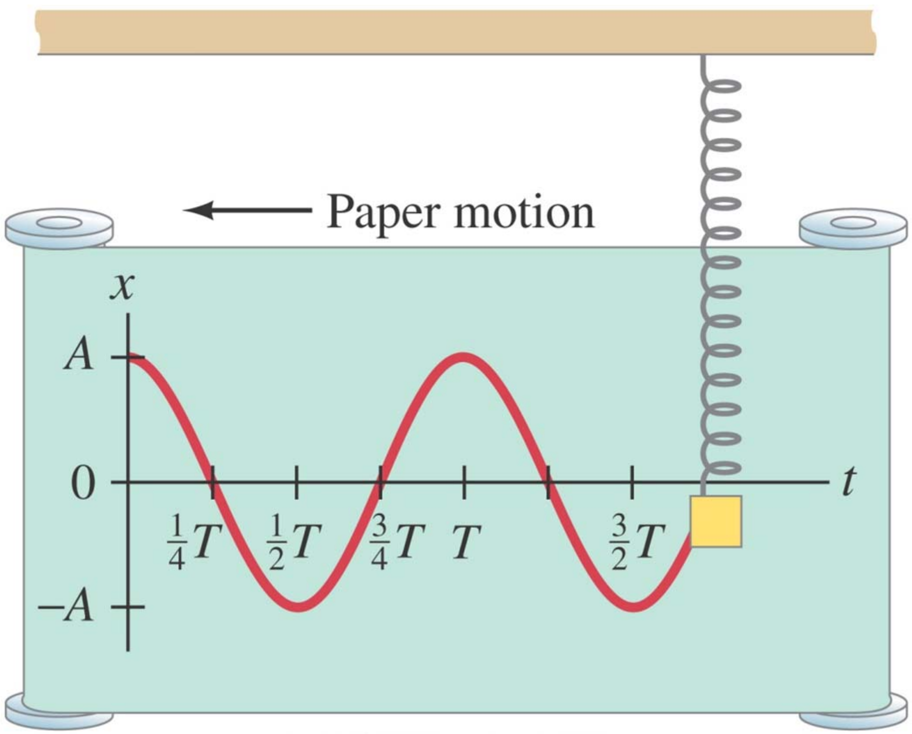
We can similarly find the position as a function of time:

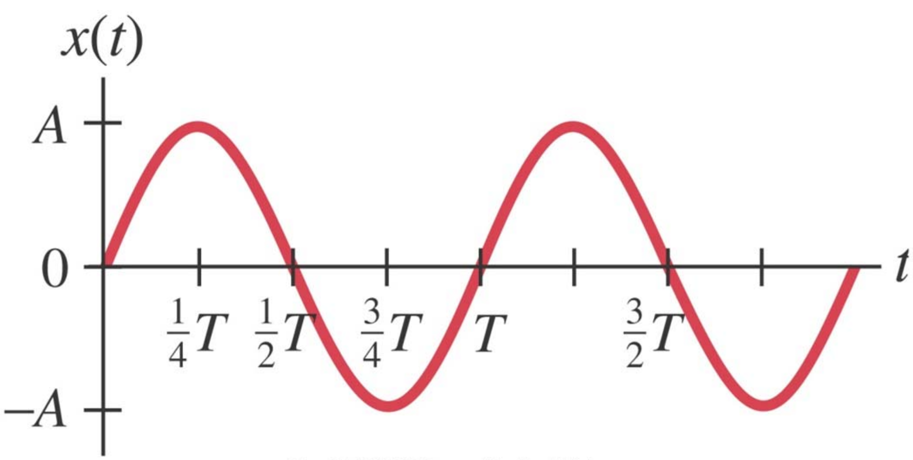
$$x = A\cos\omega t \quad \text{(11-8a)}$$

$$= A\cos(2\pi f t) \quad \text{(11-8b)}$$

$$= A\cos(2\pi t/T) \quad \text{(11-8c)}$$

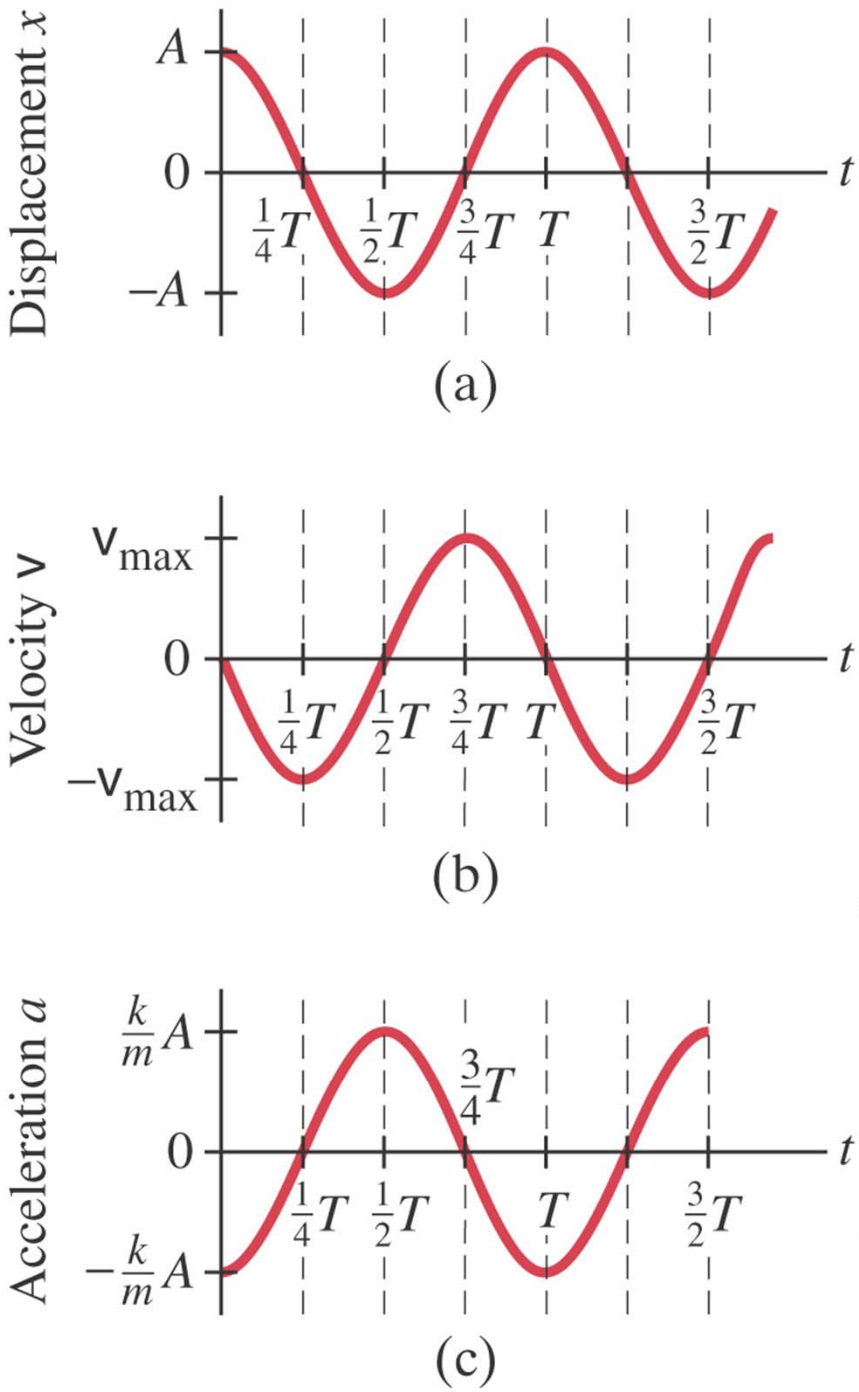
11-3 The Period and Sinusoidal Nature of SHM





The top curve is a graph of the previous equation.

The bottom curve is the same, but shifted ¼ period so that it is a sine function rather than a cosine.



11-3 The Period and Sinusoidal Nature of SHM

The velocity and acceleration can be calculated as functions of time; the results are below, and are plotted at left.

$$v = -v_{\max} \sin \omega t \qquad \textbf{(11-9)}$$

$$v_{\max} = A\sqrt{\frac{k}{m}}$$

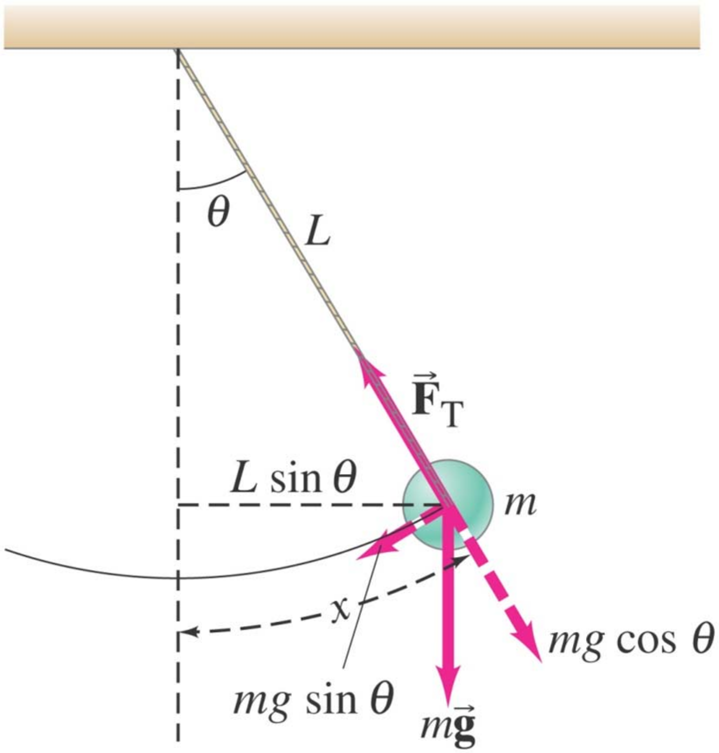
$$a = -a_{\max} \cos(2\pi t/T) \qquad \textbf{(11-10)}$$

$$a_{\max} = kA/m$$

11-4 The Simple Pendulum

A simple pendulum consists of a mass at the end of a lightweight cord. We assume that the cord does not stretch, and that its mass is negligible.

11-4 The Simple Pendulum



In order to be in SHM, the restoring force must be proportional to the displacement. Here we have:

Displacement in radians (*θ*) ***and restoring torque:***

$$\tau = mgLsin\theta$$

which is proportional to sin *θ* and not to *θ* itself.

11-4 The Simple Pendulum

However, if the angle is small, sin *θ* ≈ *θ*.

TABLE 11–1
Sin θ at Small Angles

θ (degrees)	θ (radians)	sin θ	% Difference
0	0	0	0
1°	0.01745	0.01745	0.005%
5°	0.08727	0.08716	0.1%
10°	0.17453	0.17365	0.5%
15°	0.26180	0.25882	1.1%
20°	0.34907	0.34202	2.0%
30°	0.52360	0.50000	4.7%

11-4 The Simple Pendulum

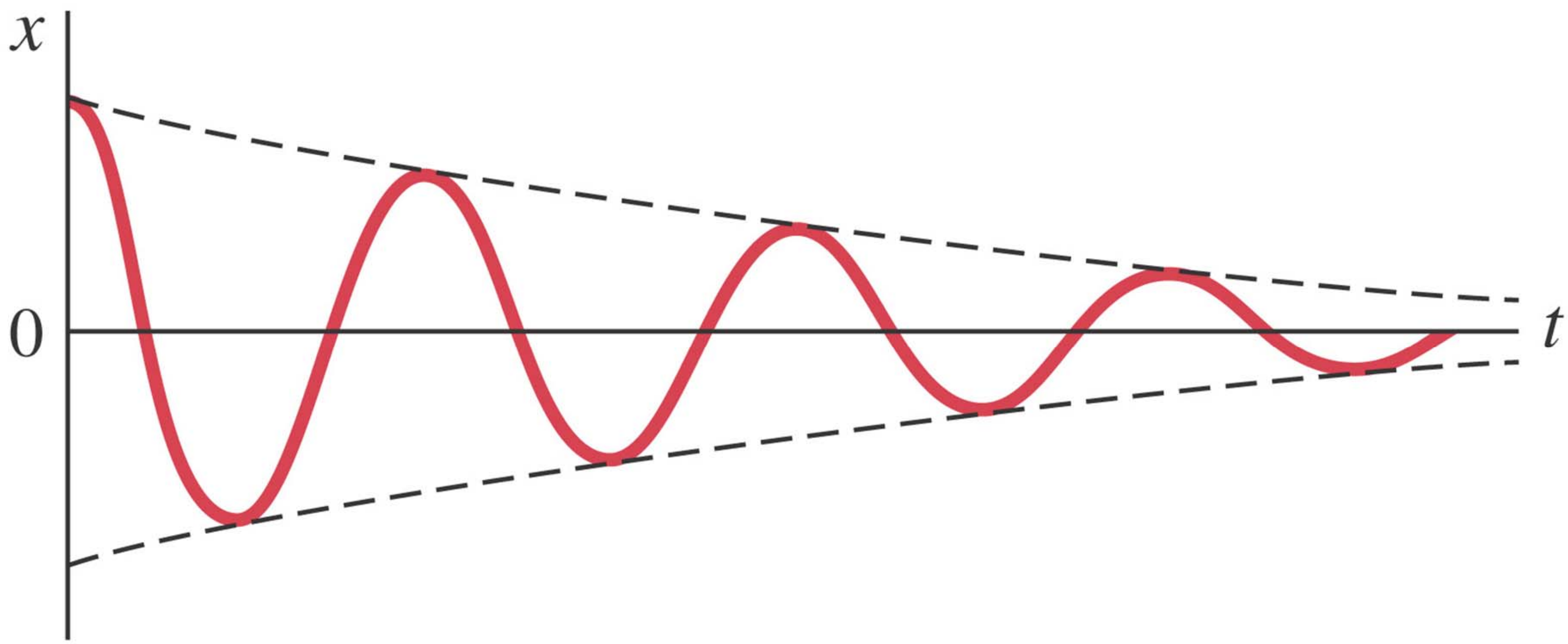
The period and frequency are:

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \text{(11-11a)}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \qquad \text{(11-11b)}$$

11-5 Damped Harmonic Motion

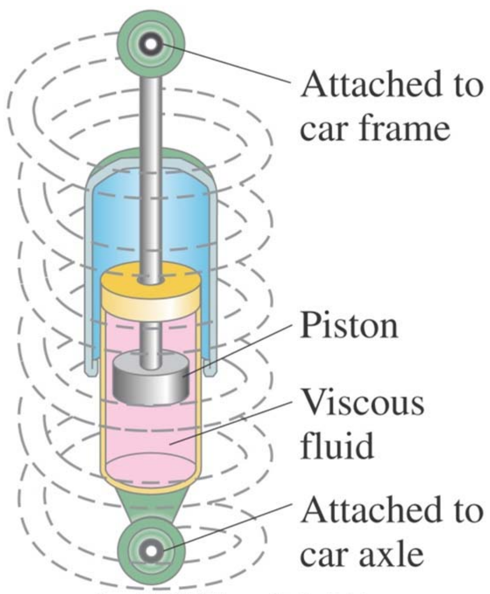
Damped harmonic motion is harmonic motion with a frictional or drag force. If the damping is small, we can treat it as an "envelope" that modifies the undamped oscillation.

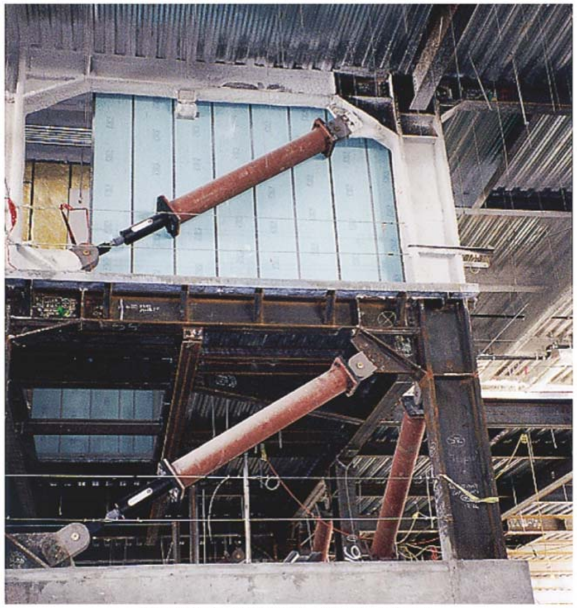


11-5 Damped Harmonic Motion

There are systems where damping is unwanted, such as clocks and watches.

Then there are systems in which it is wanted, and often needs to be as close to critical damping as possible, such as automobile shock absorbers and earthquake protection for buildings.



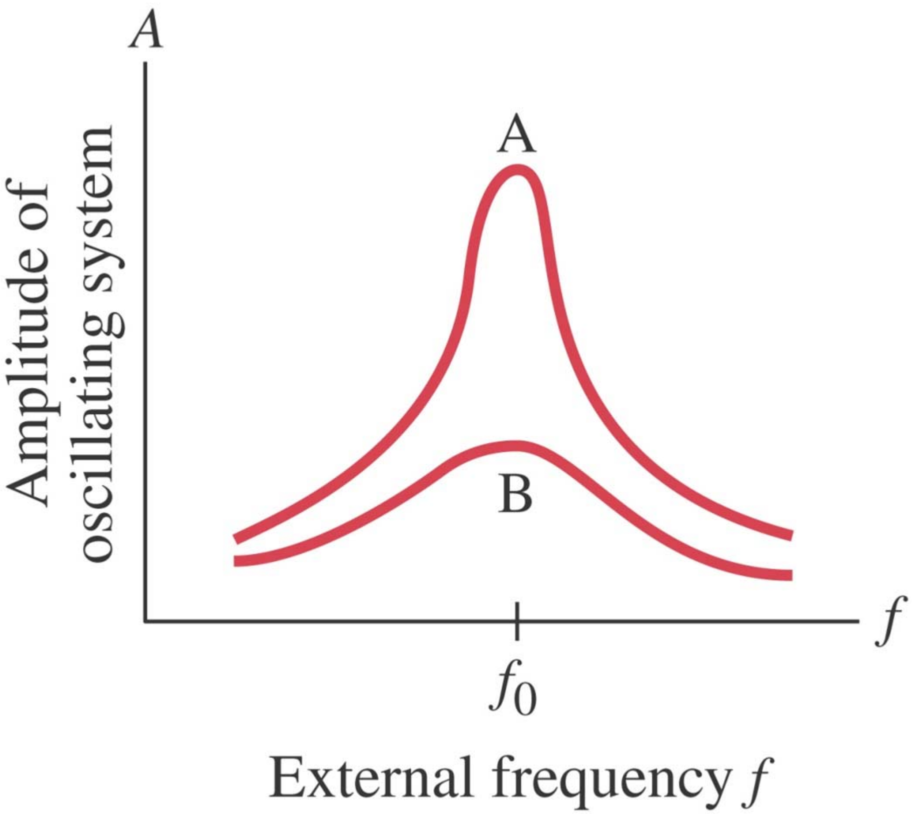


11-6 Forced Vibrations; Resonance

Forced vibrations occur when there is a periodic driving force. This force may or may not have the same period as the natural frequency of the system.

If the frequency is the same as the natural frequency, the amplitude becomes quite large. This is called resonance.

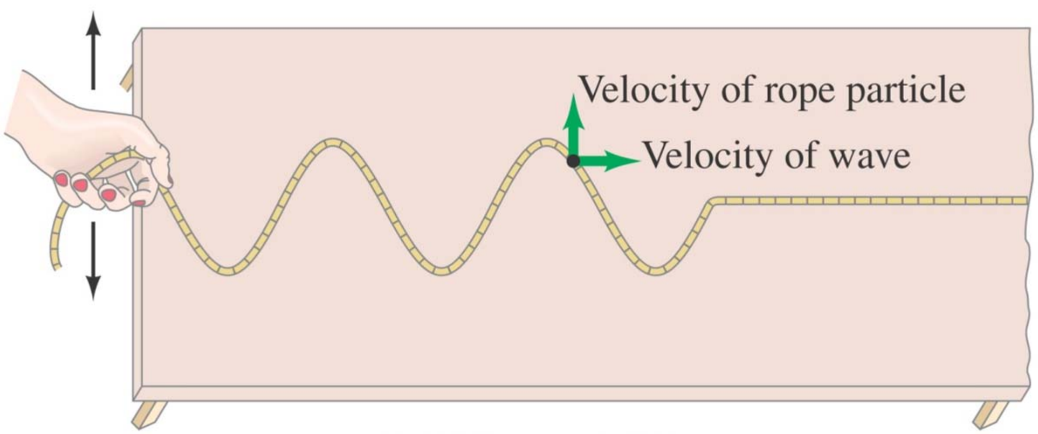
11-6 Forced Vibrations; Resonance



The sharpness of the resonant peak depends on the damping. If the damping is small (A), it can be quite sharp; if the damping is larger (B), it is less sharp.

Like damping, resonance can be wanted or unwanted. Musical instruments and TV/radio receivers depend on it.

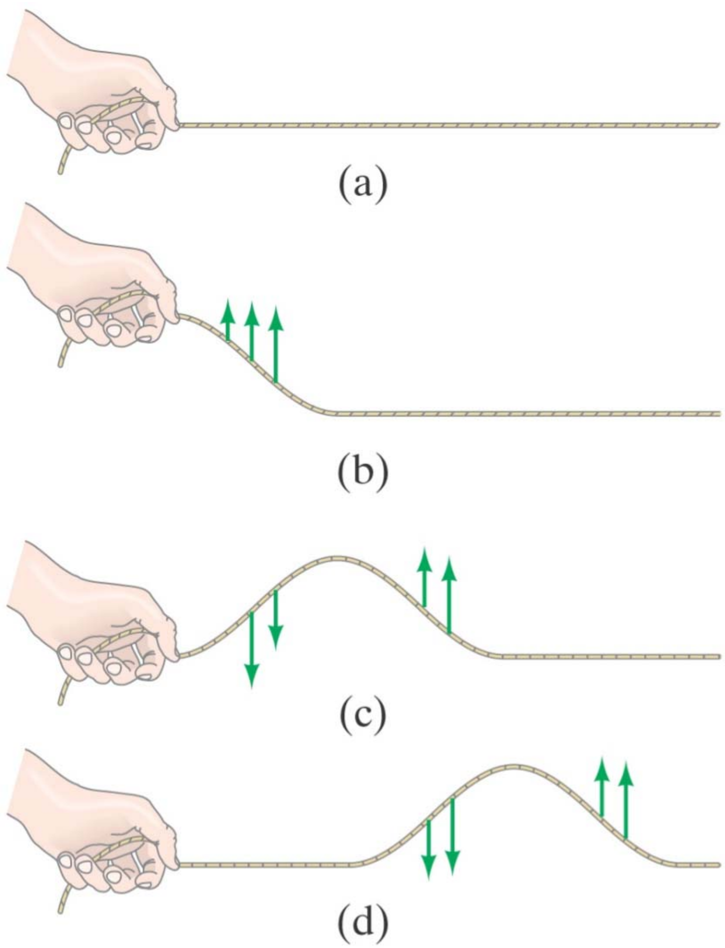
11-7 Wave Motion



A wave travels along its medium, but the individual particles just move around their equilibrium positions.

11-7 Wave Motion

All types of traveling waves transport energy.



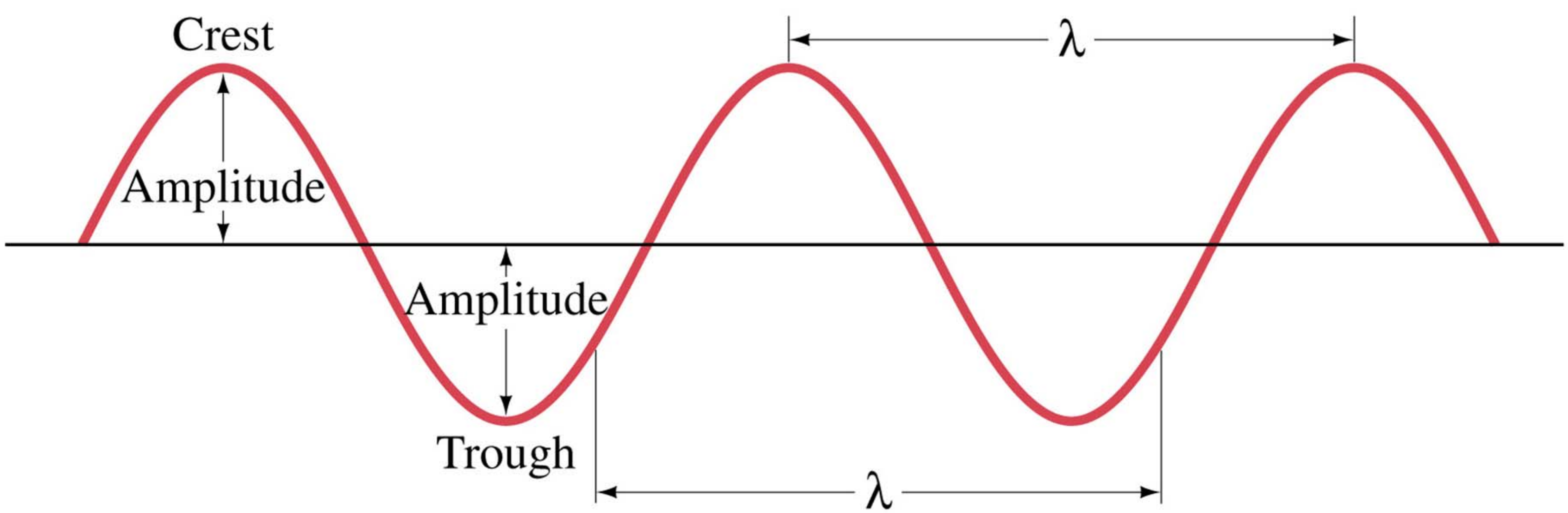
Study of a single wave pulse shows that it is begun with a vibration and transmitted through internal forces in the medium.

Continuous waves start with vibrations too. If the vibration is SHM, then the wave will be sinusoidal.

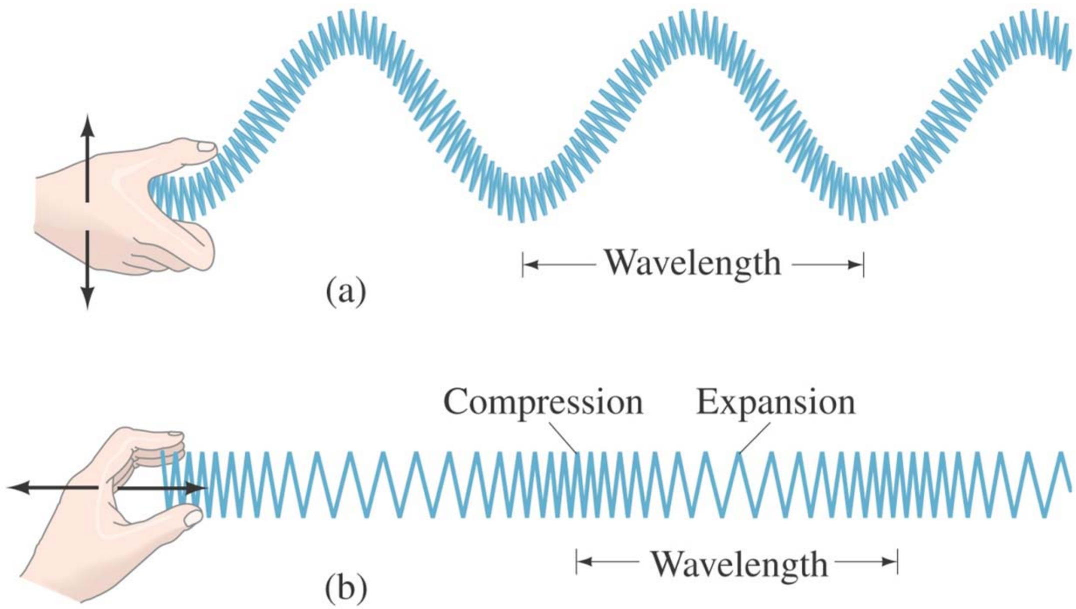
11-7 Wave Motion

Wave characteristics:

- **Amplitude, *A***
- **Wavelength, *λ***
- **Frequency *f* and period *T***
- **Wave velocity** $v = \lambda f$ **(11-12)**



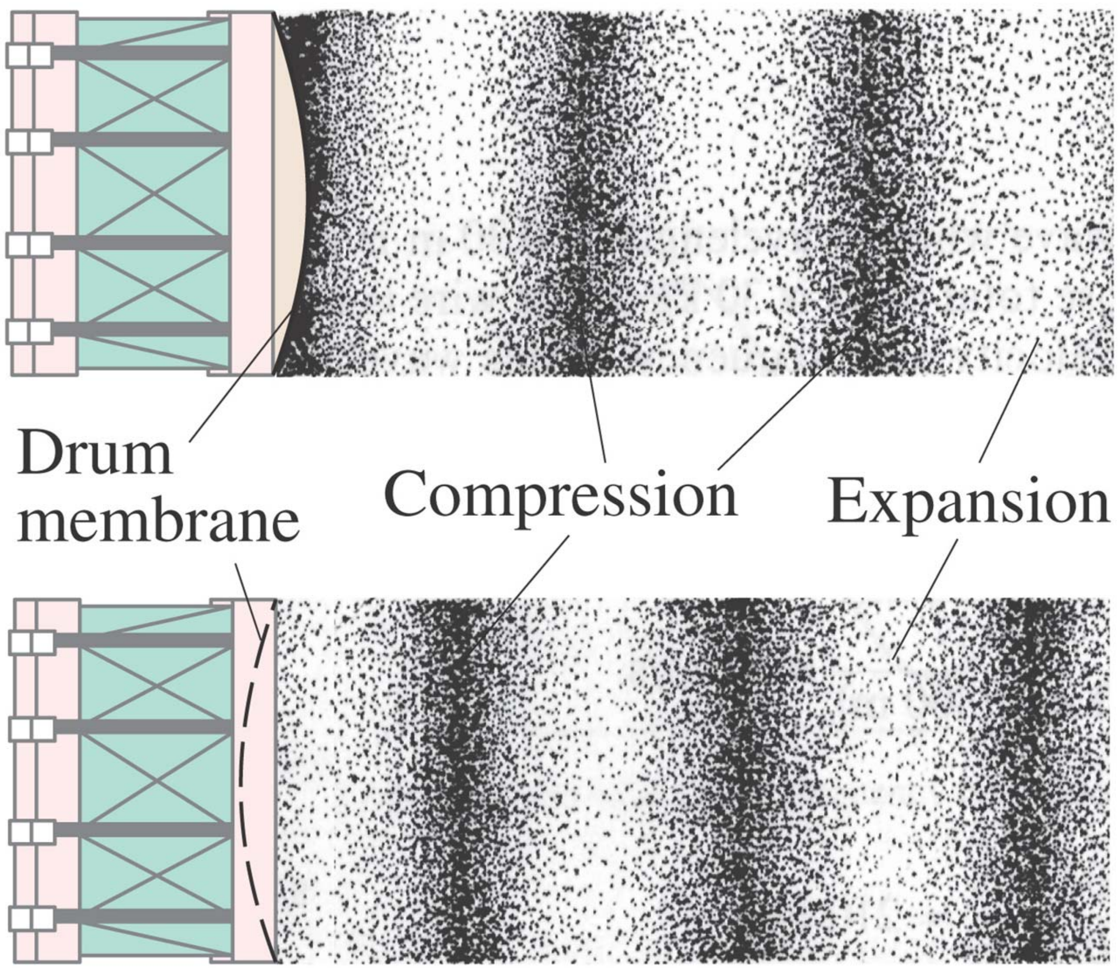
11-8 Types of Waves: Transverse and Longitudinal



The motion of particles in a wave can either be perpendicular to the wave direction (transverse) or parallel to it (longitudinal).

11-8 Types of Waves: Transverse and Longitudinal

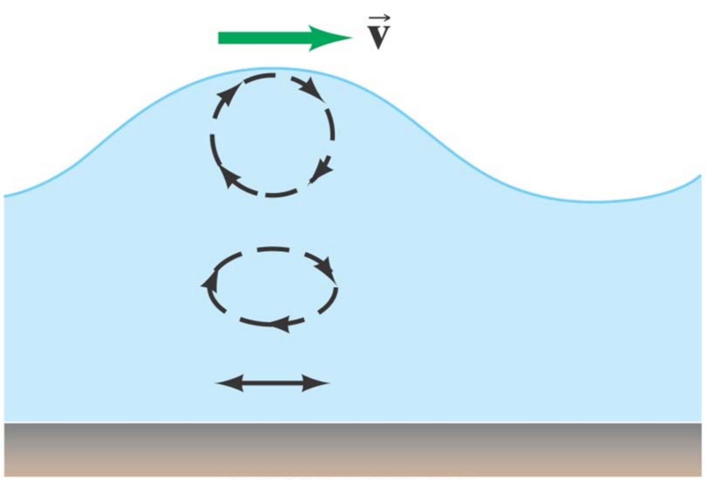
Sound waves are longitudinal waves:

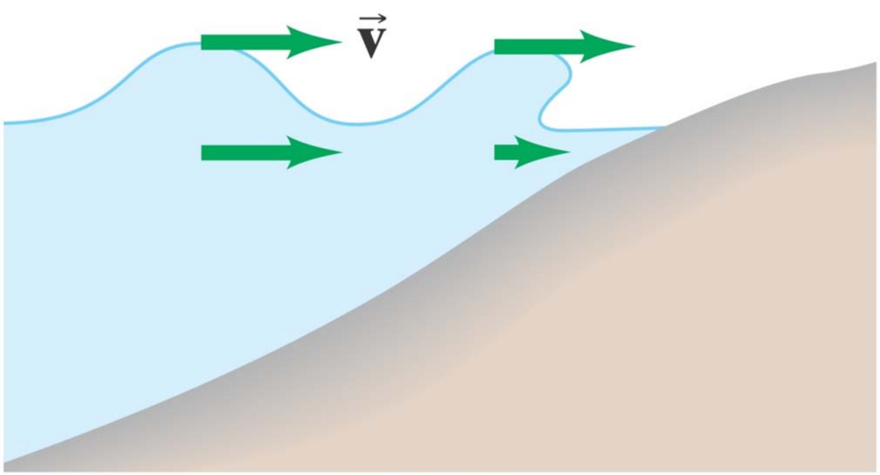


11-8 Types of Waves: Transverse and Longitudinal

Earthquakes produce both longitudinal and transverse waves. Both types can travel through solid material, but only longitudinal waves can propagate through a fluid – in the transverse direction, a fluid has no restoring force.

Surface waves are waves that travel along the boundary between two media.





11-9 Energy Transported by Waves

Just as with the oscillation that starts it, the energy transported by a wave is proportional to the square of the amplitude.

Definition of intensity:

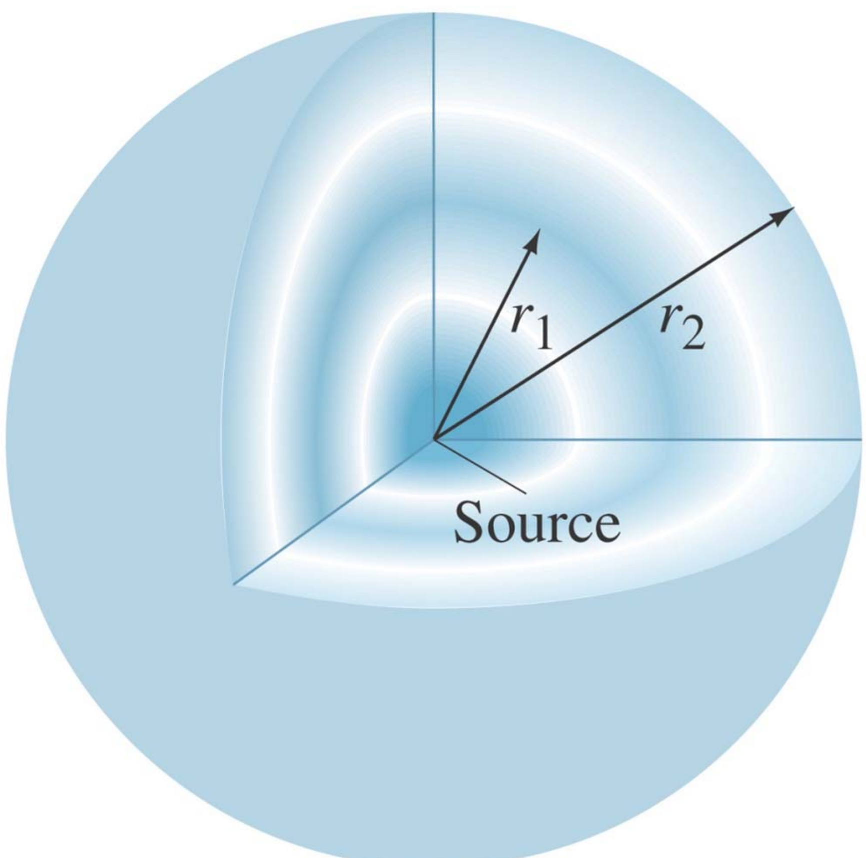
$$I = \frac{\text{energy/time}}{\text{area}} = \frac{\text{power}}{\text{area}}$$

The intensity is also proportional to the square of the amplitude:

$$I \propto A^2 \qquad \textbf{(11-15)}$$

11-9 Energy Transported by Waves

If a wave is able to spread out three-dimensionally from its source, and the medium is uniform, the wave is spherical.



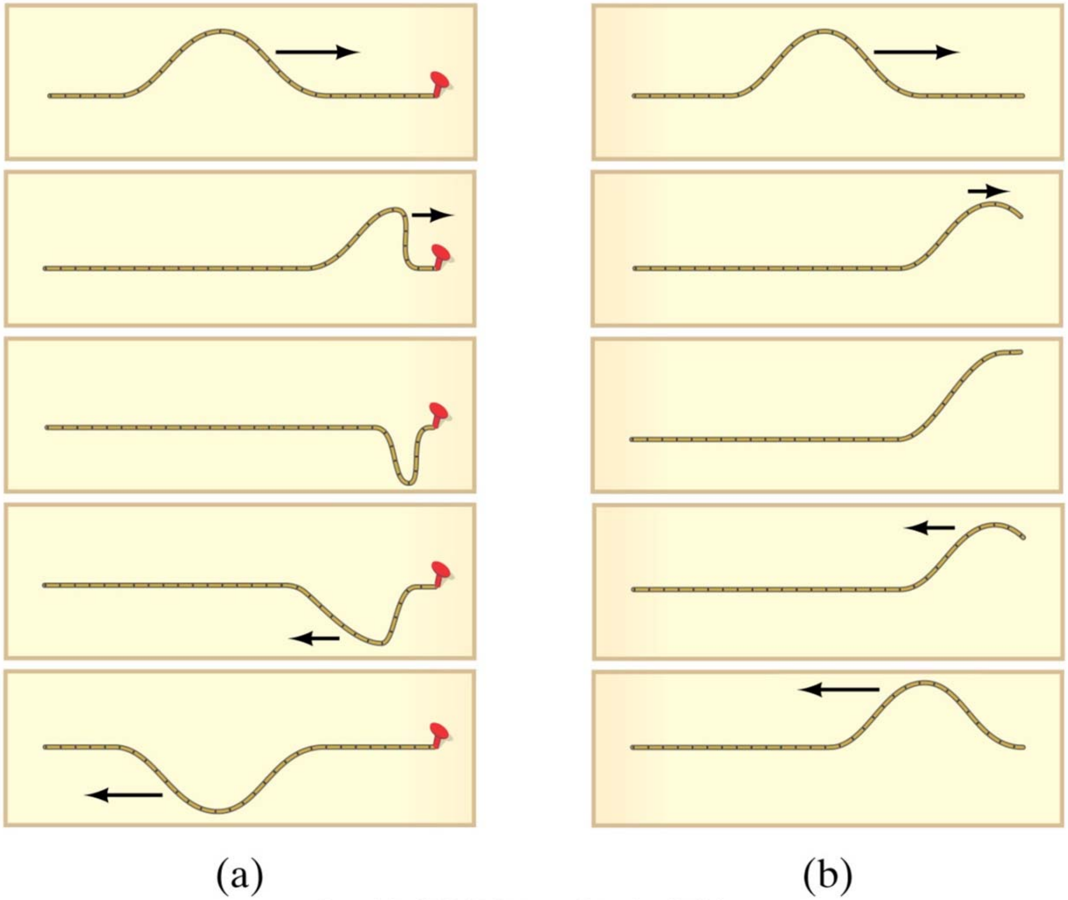
Just from geometrical considerations, as long as the power output is constant, we see:

$$I \propto \frac{1}{r^2} \quad \text{(11-16b)}$$

11-10 Intensity Related to Amplitude and Frequency

The intensity is proportional to the square of the frequency and to the square of the amplitude.

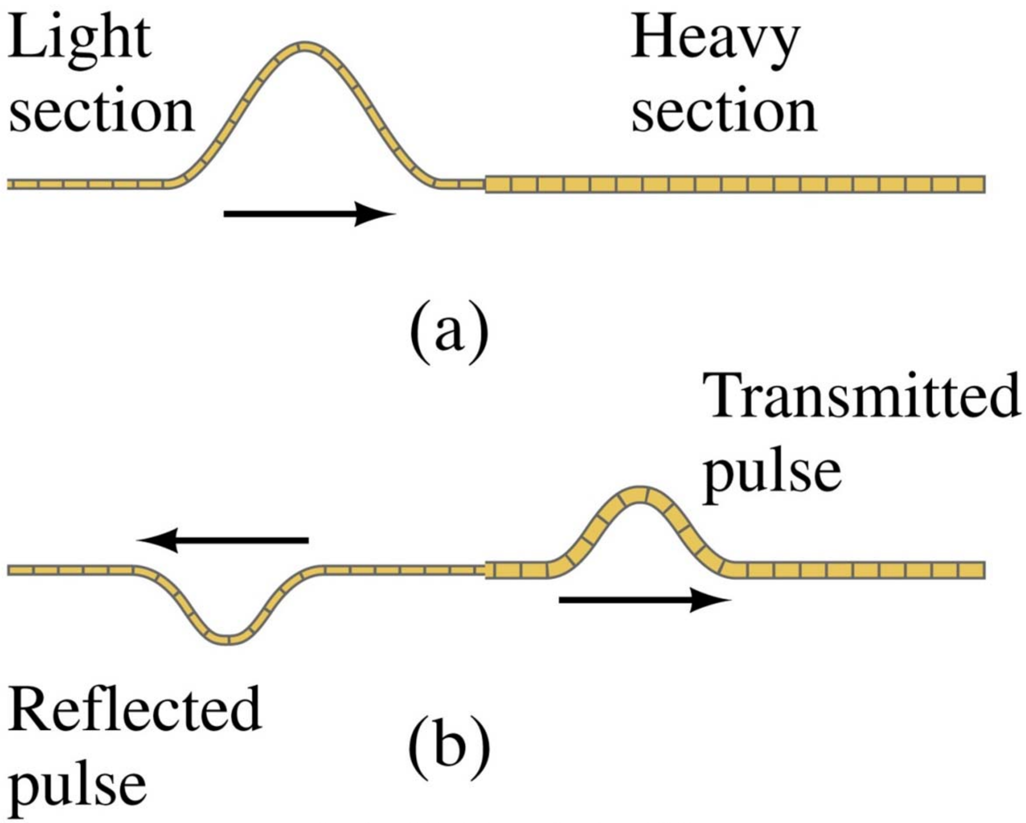
11-11 Reflection and Transmission of Waves



A wave reaching the end of its medium, but where the medium is still free to move, will be reflected (b), and its reflection will be upright.

A wave hitting an obstacle will be reflected (a), and its reflection will be inverted.

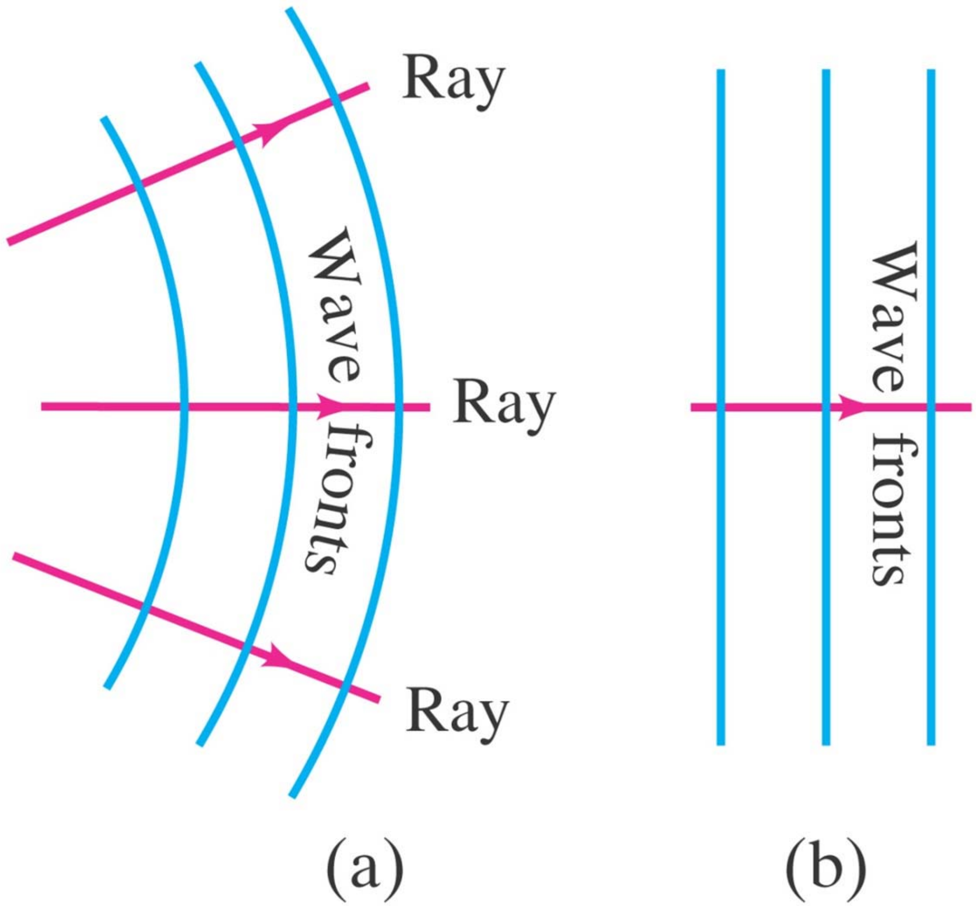
11-11 Reflection and Transmission of Waves



A wave encountering a denser medium will be partly reflected and partly transmitted; if the wave speed is less in the denser medium, the wavelength will be shorter.

11-11 Reflection and Transmission of Waves

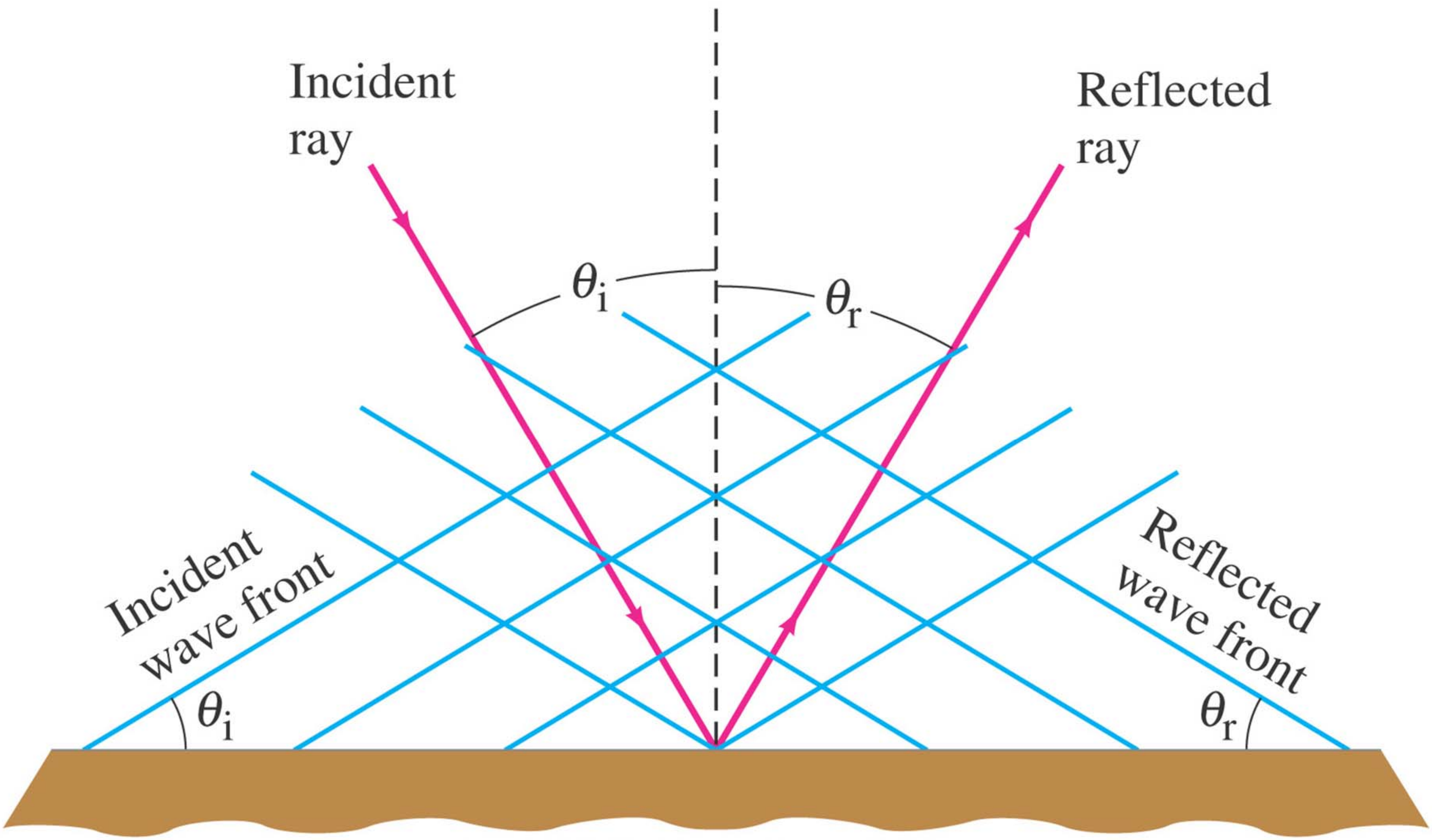
Two- or three-dimensional waves can be represented by wave fronts, which are curves of surfaces where all the waves have the same phase.



Lines perpendicular to the wave fronts are called rays; they point in the direction of propagation of the wave.

11-11 Reflection and Transmission of Waves

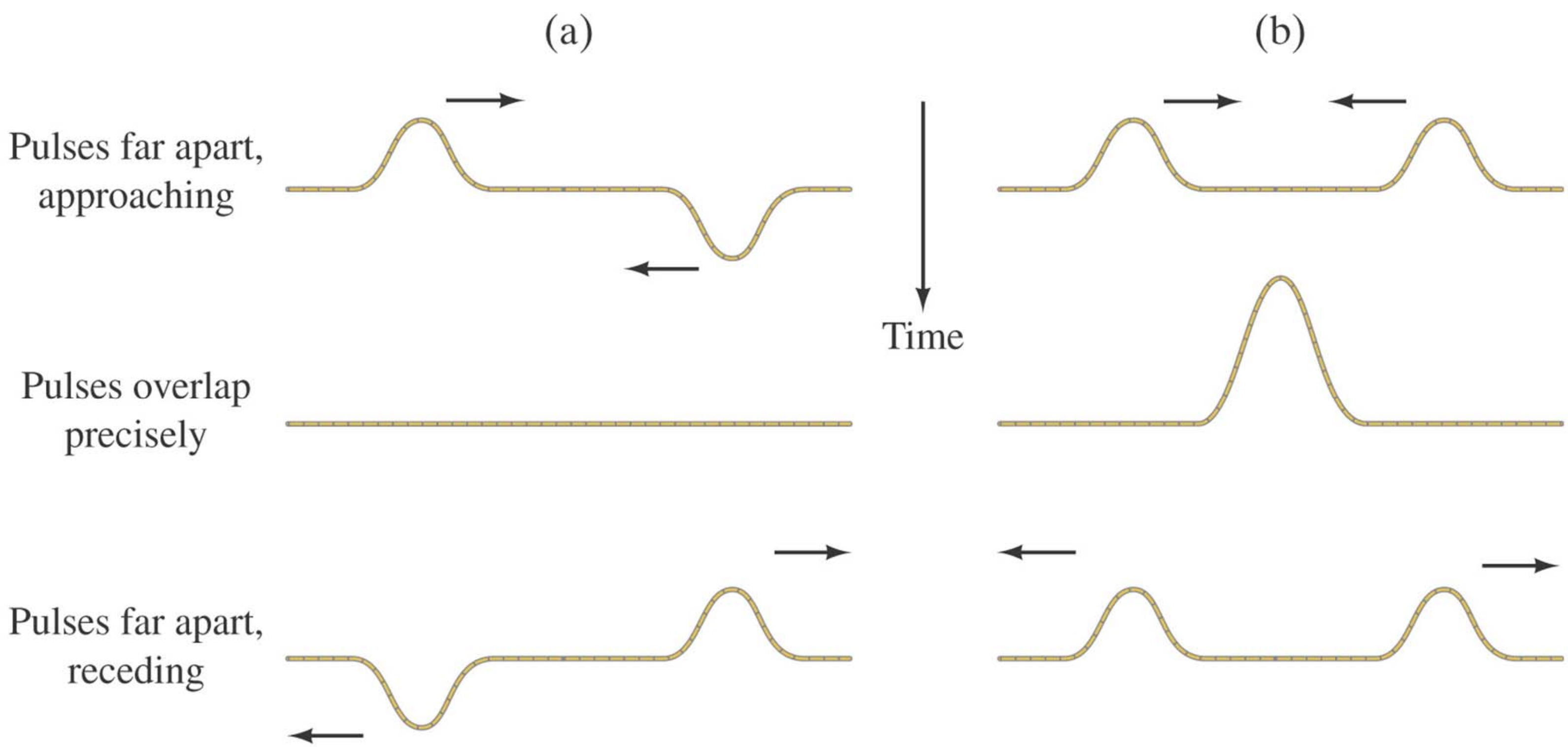
The **law of reflection**: the angle of **incidence** equals the angle of **reflection**.



11-12 Interference; Principle of Superposition

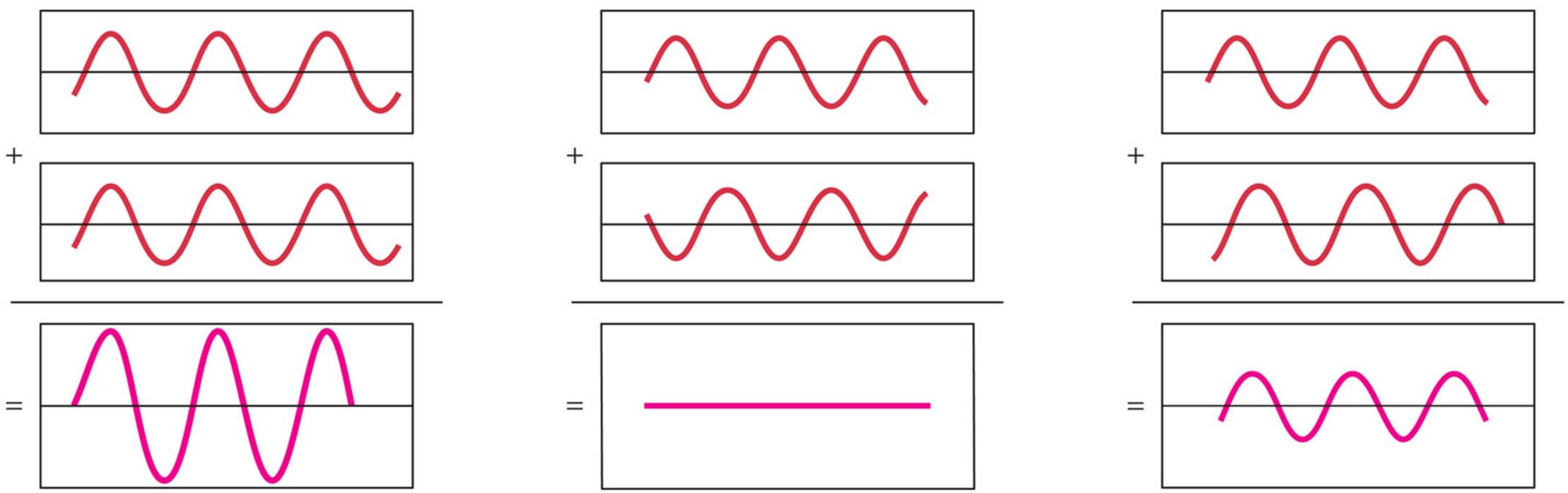
The **superposition** principle says that when two waves pass through the same point, the **displacement** is the arithmetic **sum** of the individual displacements.

In the figure below, (a) exhibits **destructive** interference and (b) exhibits **constructive** interference.



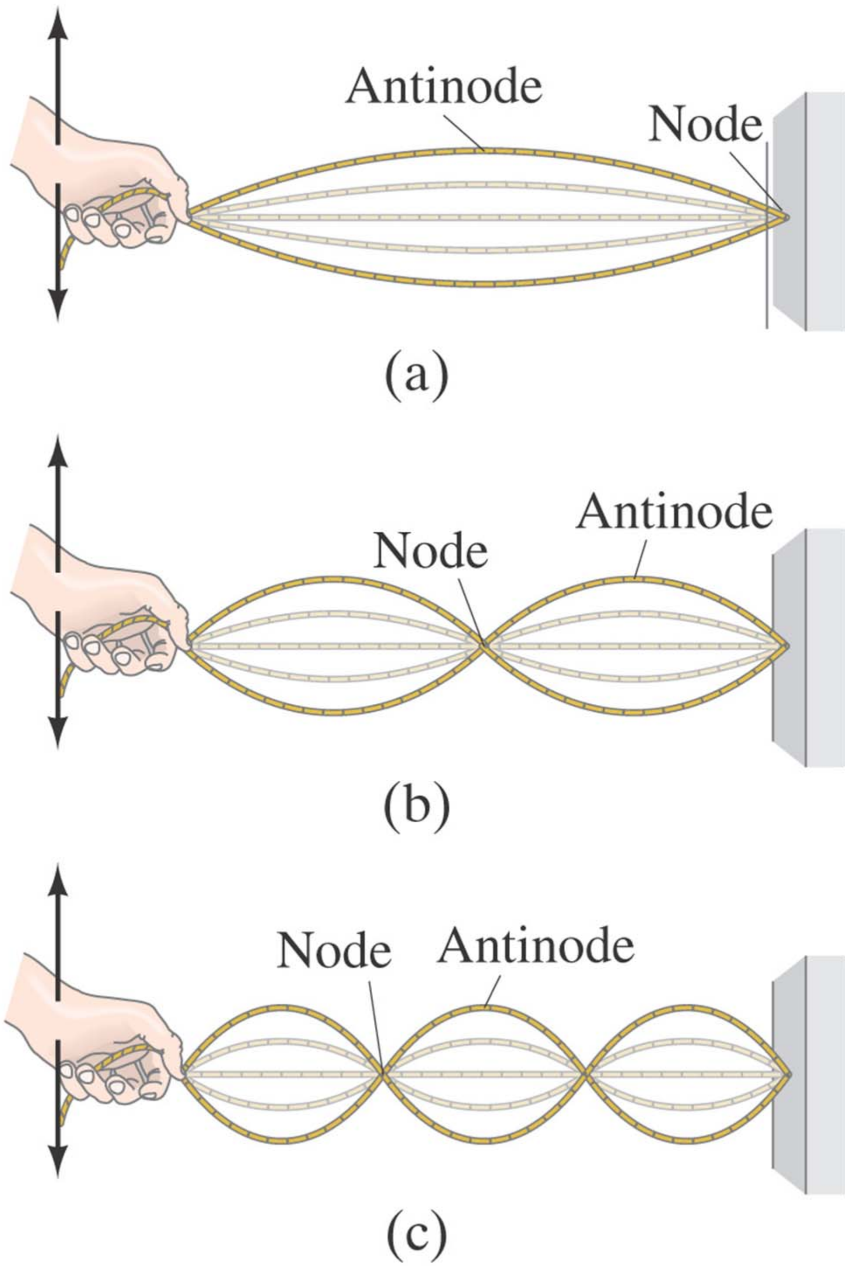
11-12 Interference; Principle of Superposition

These figures show the sum of two waves. In (a) they add **constructively**; in (b) they add **destructively**; and in (c) they add **partially destructively**.



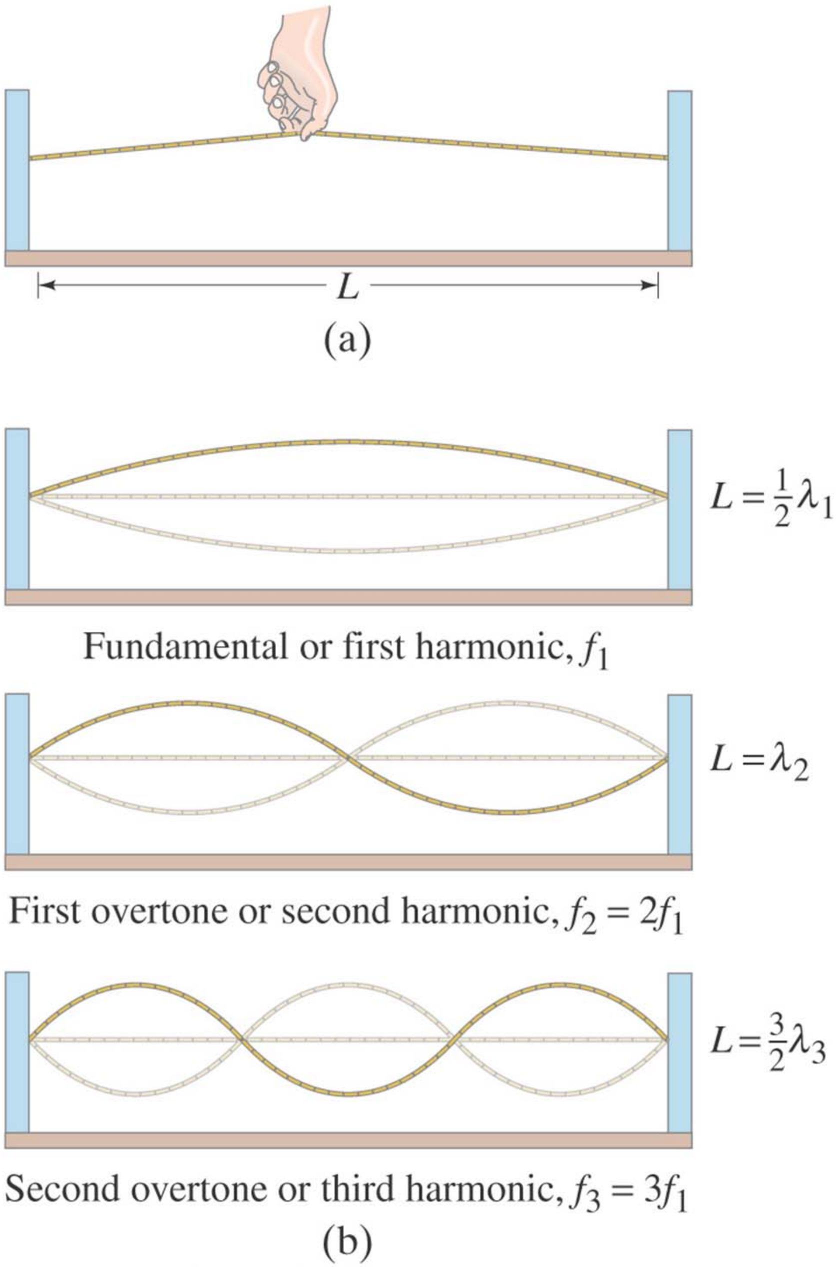
(a) (b) (c)

11-13 Standing Waves; Resonance



Standing waves occur when both ends of a string are **fixed**. In that case, only waves which are **motionless** at the ends of the string can persist. There are **nodes**, where the amplitude is always zero, and **antinodes**, where the amplitude varies from zero to the maximum value.

11-13 Standing Waves; Resonance



The **frequencies** of the standing waves on a particular string are called **resonant** frequencies.

They are also referred to as the **fundamental** and **harmonics**.

11-13 Standing Waves; Resonance

The **wavelengths** and **frequencies** of standing waves are:

$$\lambda_n = \frac{2L}{n}, \qquad n = 1, 2, 3, \cdots \qquad \text{(11-19a)}$$

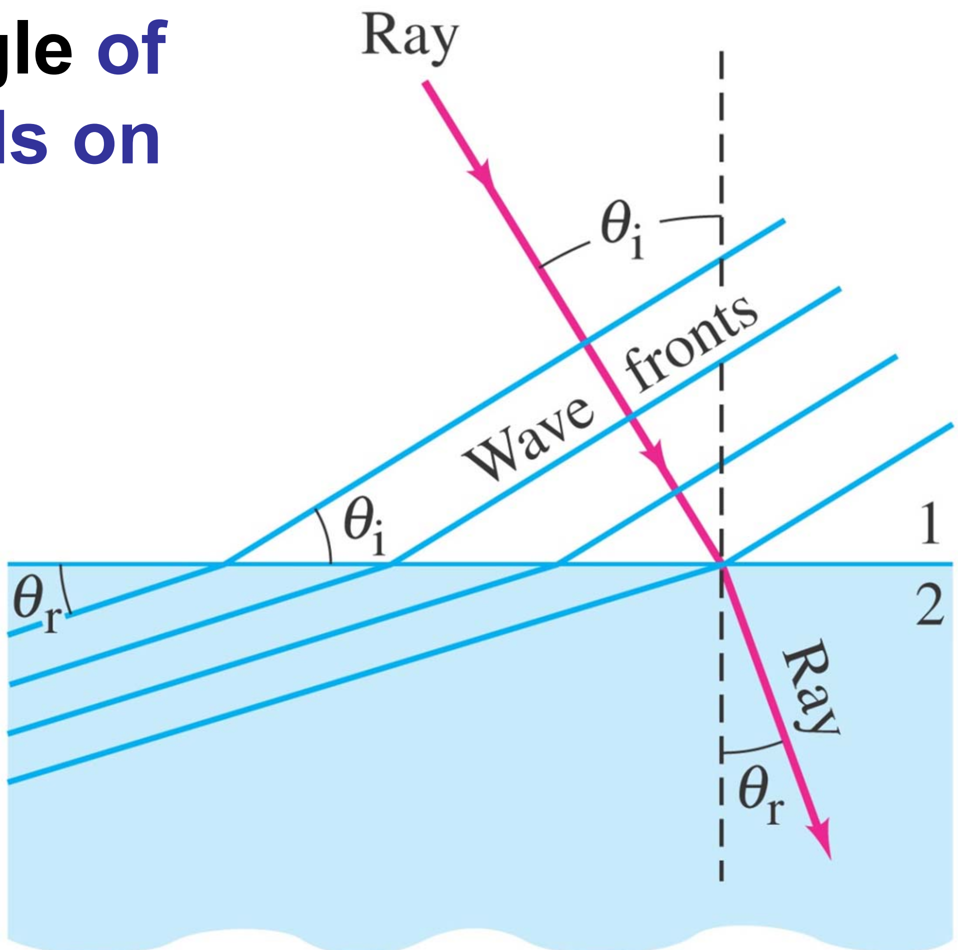
$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} = nf_1, \qquad n = 1, 2, 3, \cdots \qquad \text{(11-19b)}$$

11-14 Refraction

If the wave enters a medium where the wave speed is different, it will be refracted – its wave fronts and rays will change direction.

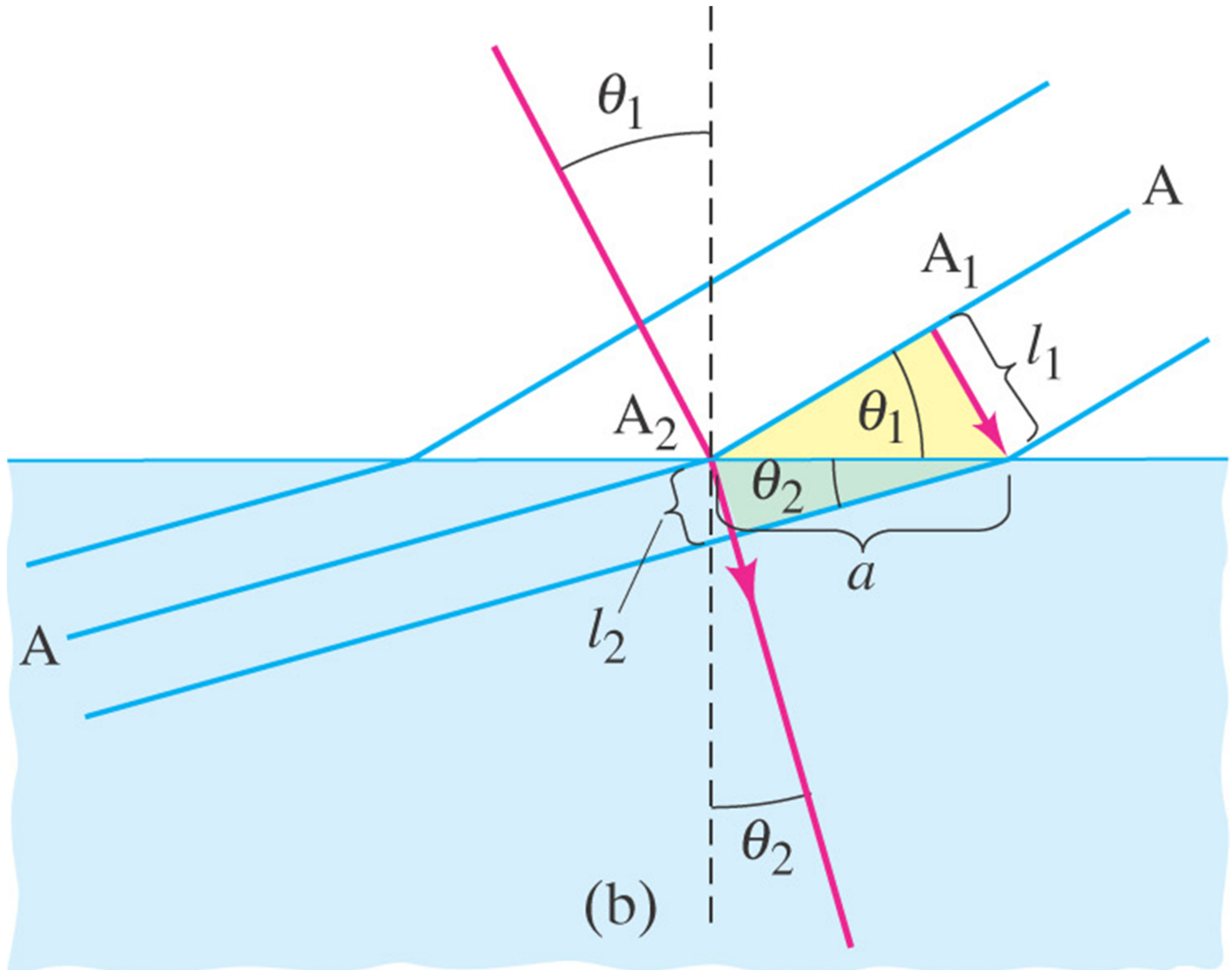
We can calculate the angle of refraction, which depends on both wave speeds:

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1} \quad \textbf{(11-20)}$$

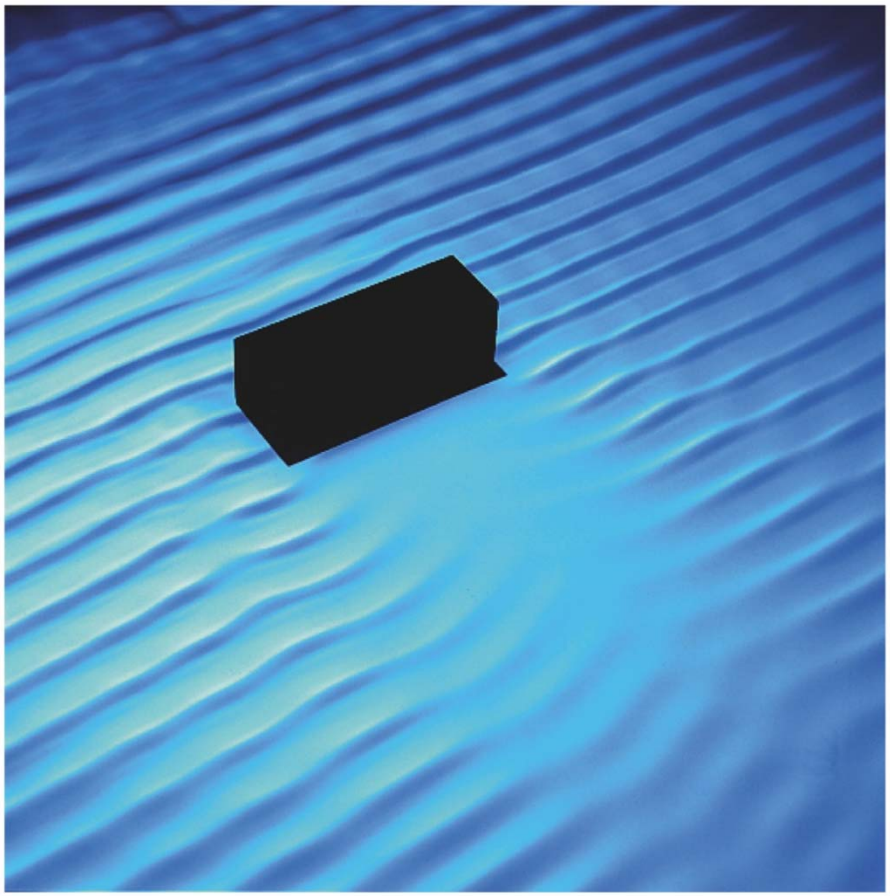


11-14 Refraction

The law of refraction works both ways – a wave going from a slower medium to a faster one would follow the red line in the other direction.



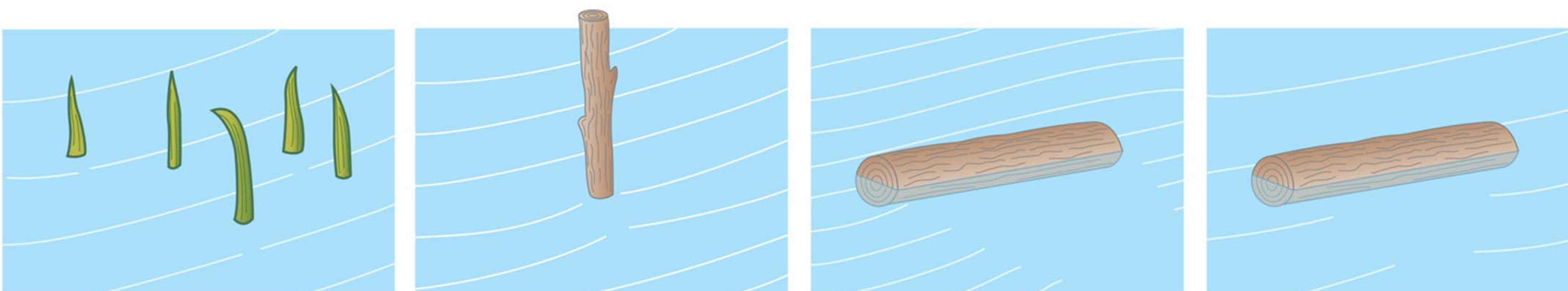
11-15 Diffraction



When waves encounter an **obstacle**, they bend around it, leaving a "shadow region." This is called **diffraction**.

11-15 Diffraction

The amount of **diffraction** depends on the size of the obstacle compared to the **wavelength**. If the obstacle is much **smaller** than the wavelength, the wave is barely affected (a). If the object is **comparable** to, or **larger** than, the wavelength, diffraction is much more significant (b, c, d).



(a) Water waves passing blades of grass (b) Stick in water (c) Short-wavelength waves passing log (d) Long-wavelength waves passing log

Summary of Chapter 11

- For SHM, the restoring force is proportional to the displacement.
- The period is the time required for one cycle, and the frequency is the number of cycles per second.
- Period for a mass on a spring: $$T = 2\pi\sqrt{\frac{m}{k}}$$
- SHM is sinusoidal.
- During SHM, the total energy is continually changing from kinetic to potential and back.

Summary of Chapter 11

- A simple pendulum approximates SHM if its amplitude is not large. Its period in that case is:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

- When friction is present, the motion is damped.

- If an oscillating force is applied to a SHO, its amplitude depends on how close to the natural frequency the driving frequency is. If it is close, the amplitude becomes quite large. This is called resonance.

Summary of Chapter 11

- Vibrating objects are sources of waves, which may be either a pulse or continuous.
- Wavelength: distance between successive crests.
- Frequency: number of crests that pass a given point per unit time.
- Amplitude: maximum height of crest.
- Wave velocity: $v = \lambda f$

Summary of Chapter 11

- Vibrating objects are sources of waves, which may be either a pulse or continuous.
- Wavelength: distance between successive crests
- Frequency: number of crests that pass a given point per unit time
- Amplitude: maximum height of crest
- Wave velocity: $v = \lambda f$

Summary of Chapter 11

• Transverse wave: oscillations perpendicular to direction of wave motion.

• Longitudinal wave: oscillations parallel to direction of wave motion.

• Intensity: energy per unit time crossing unit area (W/m^2):

$$I \propto \frac{1}{r^2}$$

• Angle of reflection is equal to angle of incidence.

Summary of Chapter 11

- When two waves pass through the same region of space, they interfere. Interference may be either constructive or destructive.

- Standing waves can be produced on a string with both ends fixed. The waves that persist are at the resonant frequencies.

- Nodes occur where there is no motion; antinodes where the amplitude is maximum.

- Waves refract when entering a medium of different wave speed, and diffract around obstacles.